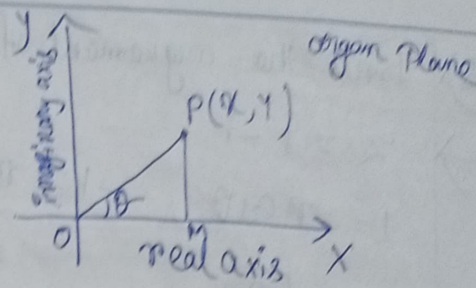
Complex number

$z = x + iy$

$|z| = \sqrt{x^2+y^2} = OP$

$\arg(z) = \tan^{-1}\frac{y}{x} = \theta$

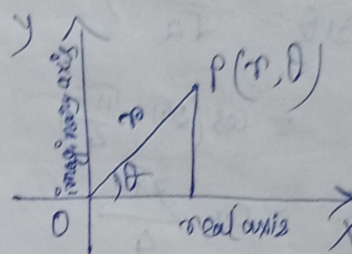


Polar representation:–

$x = r\cos\theta, \quad y = r\sin\theta$

$\therefore x^2+y^2 = r^2$

or, $r = \sqrt{x^2+y^2} = |z|$



~~Polar representation:–~~

$\therefore \tan\theta = \frac{y}{x}$

or, $\theta = \tan^{-1}\frac{y}{x} = \arg(z)$

$\therefore z = r(\cos\theta + i\sin\theta)$

Note:– If θ be the arguement of the complex number $z = x+iy$

$-\pi < \theta \le \pi$.

1) Find the modulus and argument of the complex number $-1-i$.

$\Rightarrow z = -1-i$

Let, $-1 = r\cos\theta$ and $-1 = r\sin\theta$

$\therefore r = \sqrt{1^2+1^2} = \sqrt{2}$

$\therefore \cos\theta = -\frac{1}{\sqrt{2}}, \quad \sin\theta = -\frac{1}{\sqrt{2}}$

$\cancel{\therefore \tan\theta = \frac{-\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = \tan^{-1}(-1)}$

$= -\cos\frac{\pi}{4}$ $\qquad = -\sin\frac{\pi}{4}$

$= \cos\left(\pi + \frac{\pi}{4}\right)$ $\qquad = \sin\left(\frac{\pi}{4} + \pi\right)$

$= \cos\frac{5\pi}{4}$ $\qquad = \sin\frac{5\pi}{4}$

$\cancel{\theta = \pi - \frac{5\pi}{4} = -\frac{\pi}{4}}$

$\cancel{\therefore \theta = \pi - \frac{5\pi}{4} = -\frac{3\pi}{4}}$

$\cancel{\therefore \theta = \frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}}$

$\cancel{\therefore \theta = \pi - \frac{5\pi}{4} = -\frac{\pi}{4}}$

$\therefore \theta = \frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}$

2) Find the arguement of the complex number $z = 1 - i$

$\Rightarrow$ $z = 1 - i$

$1 = r\cos\theta$, $-1 = r\sin\theta$

$\therefore r = \sqrt{2}$

$\therefore \cos\theta = \frac{1}{\sqrt{2}}$, $\sin\theta = -\frac{1}{\sqrt{2}}$

$= \cos\left(2\pi - \frac{\pi}{4}\right)$ $\quad = \sin\left(2\pi - \frac{\pi}{4}\right)$

$= \cos\frac{7\pi}{4}$ $\quad = \sin\frac{7\pi}{4}$

$\therefore \theta = \frac{7\pi}{4} - 2\pi = \frac{7\pi - 8\pi}{4} = -\frac{\pi}{4}$

$\therefore \arg|z| = -\frac{\pi}{4}$

3) Show that $(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)\ldots(\cos\theta_n + i\sin\theta_n)$
$= \cos(\theta_1 + \theta_2 + \ldots + \theta_n) + i\sin(\theta_1 + \theta_2 + \ldots + \theta_n)$

$\Rightarrow$ We have,

$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2) = (\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1)$

$= \cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)$ —— (i)

$\therefore$ The result is true for $n = 2$.

Let, the result be true for $n = m$.

$\therefore (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)\ldots(\cos\theta_m + i\sin\theta_m) = \cos(\theta_1 + \theta_2 + \ldots + \theta_m) + i\sin(\theta_1 + \theta_2 + \ldots + \theta_m)$

Now we have,

$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)\ldots(\cos\theta_m + i\sin\theta_m)(\cos\theta_{m+1} + i\sin\theta_{m+1})$

$= \{\cos(\theta_1 + \theta_2 + \ldots + \theta_m) + i\sin(\theta_1 + \theta_2 + \ldots + \theta_m)\}(\cos\theta_{m+1} + i\sin\theta_{m+1})$

$= \cos(\theta_1 + \theta_2 + \ldots + \theta_m + \theta_{m+1}) + i\sin(\theta_1 + \theta_2 + \ldots + \theta_m + \theta_{m+1})$

[by (i)]

This shows that the result is true for $n = m + 1$.

$\therefore$ By the principle of mathematical induction

$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)\ldots(\cos\theta_n + i\sin\theta_n) = \cos(\theta_1 + \theta_2 + \ldots + \theta_n) + i\sin(\theta_1 + \theta_2 + \ldots + \theta_n)$

4) If $z_n = \cos\frac{\pi}{3^n} + i\sin\frac{\pi}{3^n}$, $n = 1, 2, 3, \ldots$ then show that $z_1 z_2 z_3 \ldots \infty = i$

$\Rightarrow$ $z_1 z_2 z_3 \ldots \infty$

$= \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{3^2} + i\sin\frac{\pi}{3^2}\right)\left(\cos\frac{\pi}{3^3} + i\sin\frac{\pi}{3^3}\right)\ldots$

$= \cos\left(\frac{\pi}{3} + \frac{\pi}{3^2} + \frac{\pi}{3^3} + \ldots\right) + i\sin\left(\frac{\pi}{3} + \frac{\pi}{3^2} + \frac{\pi}{3^3} + \ldots\right)$

$= \cos\frac{\pi}{3}\left(1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\right) + i\sin\frac{\pi}{3}\left(1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\right)$

$= \cos\frac{\pi}{3}\frac{1}{1-\frac{1}{3}} + i\sin\frac{\pi}{3}\,6\,\frac{1}{1-3} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$.

5) **D'Moivre's Theorem :-**

If n is integer, positive or negative then $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ and if n is fractional (Positive or negative) then one of the values of $(\cos\theta + i\sin\theta)^n$ is $\cos n\theta + i\sin n\theta$.

Proof:-

Case-1:- Let, n be positive integer.

We have, $(\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta = \cos(1\cdot\theta) + i\sin(1\cdot\theta)$

$\therefore$ The result is true for $n = 1$

and $(\cos\theta + i\sin\theta)^2 = \cos^2\theta - \sin^2\theta + 2i\sin\theta\cos\theta$

$= \cos 2\theta + i\sin 2\theta$

$\therefore$ The result is also true for $n = 2$.

~~$\therefore$ The~~

Now, Let, it be true for $n = m$.

$\therefore (\cos\theta + i\sin\theta)^m = \cos m\theta + i\sin m\theta$ —(i)

Then,

$\therefore (\cos\theta + i\sin\theta)^{m+1} = (\cos\theta + i\sin\theta)^m \cdot (\cos\theta + i\sin\theta)$

$= (\cos m\theta + i\sin m\theta)(\cos\theta + i\sin\theta)$ [by (i)]

$= \cos(m\theta + \theta) + i\sin(m\theta + \theta)$

$= \cos(m+1)\theta + i\sin(m+1)\theta$

$\therefore$ The result is true for $n = m+1$

$\therefore$ By the principle of mathematical induction $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ for all positive integral value of n.

Case - II :- Let, n be negative integer and $n = -p$ where p is positive integer.

$$\therefore (\cos\theta + i\sin\theta)^n = (\cos\theta + i\sin\theta)^{-p}$$

$$= \frac{1}{(\cos\theta + i\sin\theta)^p} = \frac{1}{\cos p\theta + i\sin p\theta} \quad [\text{by Case I}]$$

$$= \cos p\theta - i\sin p\theta = \cos(-p\theta) + i\sin(-p\theta)$$

$$= \cos n\theta + i\sin n\theta.$$

$\therefore$ The theorem is true for all negative integer n.

Case-III :- Let, n be fraction,

Let, $n = \frac{p}{q}$, where p and q are both integers and $q \neq 0$,

We have,

$$\cos\theta + i\sin\theta = \left(\cos\frac{\theta}{q} + i\sin\frac{\theta}{q}\right)^q$$

$\therefore$ one of the values of $(\cos\theta + i\sin\theta)^{1/q}$ is $\left(\cos\frac{\theta}{q} + i\sin\frac{\theta}{q}\right)$

$\therefore$ One of the values of $(\cos\theta + i\sin\theta)^{p/q}$ is $\left(\cos\frac{\theta}{q} + i\sin\frac{\theta}{q}\right)^p$

$$= \cos\frac{p}{q}\theta + i\sin\frac{p}{q}\theta$$

$\therefore$ One of the values of $(\cos\theta + i\sin\theta)^n$ is $(\cos n\theta + i\sin n\theta)$

6) Find all the values of $1^{1/3}$.

$\Rightarrow$ $1 = \cos\theta + i\sin\theta = (\cos 2k\pi + i\sin 2k\pi)$, $k \in \mathbb{Z}$

$$\therefore 1^{1/3} = (\cos 2k\pi + i\sin 2k\pi)^{1/3}, \quad k = 0, 1, 2.$$

$$= \cos\frac{2k\pi}{3} + i\sin\frac{2k\pi}{3}, \quad k = 0, 1, 2.$$

$\therefore$ The values are $\cos 0 + i\sin 0$, $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$ and $\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

$$= 1, \; -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \; -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$

$$= 1, \omega, \omega^2.$$

1) Find the values of $(-i)^{3/5}$.

→ We have,

$$-i = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}$$

$$(-i)^3 = \left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)^3 = \cos\frac{9\pi}{2} + i\sin\frac{9\pi}{2}$$

$$= \cos\left(2k\pi + \frac{9\pi}{2}\right) + i\sin\left(2k\pi + \frac{9\pi}{2}\right), \quad k \in \mathbb{Z}$$

$$\therefore (-i)^{3/5} = \left(\cos\frac{\left(2k\pi + \frac{9\pi}{2}\right)}{5} + i\sin\frac{2k\pi + \frac{9\pi}{2}}{5}\right), \quad k = 0, 1, 2, 3, 4.$$

$\therefore$ There are 5 distinct values and they are —

$$\left(\cos\frac{9\pi}{10} + i\sin\frac{9\pi}{10}\right), \left(\cos\frac{13\pi}{10} + i\sin\frac{13\pi}{10}\right), \left(\cos\frac{17\pi}{10} + i\sin\frac{17\pi}{10}\right), \left(\cos\frac{21\pi}{10} + i\sin\frac{21\pi}{10}\right),$$

$$\left(\cos\frac{25\pi}{10} + i\sin\frac{25\pi}{10}\right)$$

Theory of Equation

1) **Remainder Theorem :-** When a Polynomial $f(x)$ is divided by $x-h$ then the remainder is $f(h)$.

2) Find the remainder when x^3+5x^2+3x+2 is divided by $x-1$

$\Rightarrow$ Let, $f(x) = x^3+5x^2+3x+2$

Remainder is $f(1) = 1+5+3+2 = 11$

Synthetic method :-

3) Find the quotient and the remainder when $x^4+5x^3+4x^2+8x-2$ is divided by $x+2$.

$\Rightarrow$

-2	1	5	4	8	-2
		-2	-6	4	-24
	1	3	-2	12	-26

$x^3+3x^2-2x+12$

$\therefore$ The quotient is, $Q(x) = x^3+3x^2-2x+12$ and the remainder $R(x) = -26$.

4) **Root :-** α is said to be a root of the equation $f(x)=0$ if $f(\alpha)=0$.

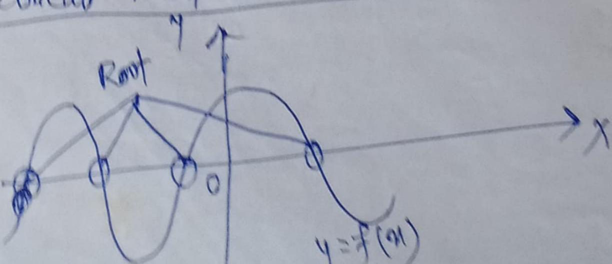
[Note :- If α is a root of the polynomial equation $f(\alpha)=0$ then $x-\alpha$ is a factor of $f(x)$.]

5) **Fundamental theorem of classical Algebra :-**

Every algebraic equation has atleast one root, real or imaginary.

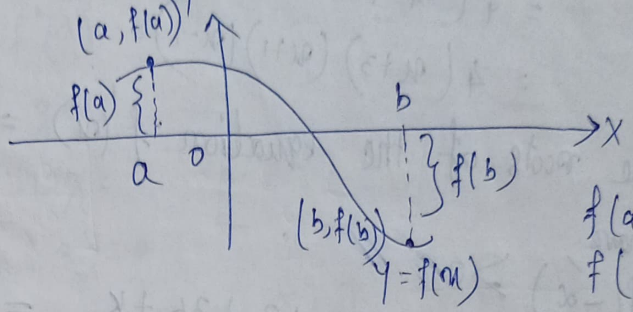
[Note :- i) Every n degree Polynomial equation has exactly n roots.
ii) If $\alpha_1, \alpha_2, \alpha_n$ are the roots of the polynomial equation $f(x)=0$ then $f(x) = (x-\alpha_1)(x-\alpha_2)\ldots(x-\alpha_n)$]

6) **Geometrical interpretation of roots :-**



The points where the curve $y=f(x)$ meets the axis of 'X' are the roots of the equation $f(x)=0$.

7) Bolzano's Theorem:- If $f(a)$ and $f(b)$ are of opposite signs $(f(a)\cdot f(b)<0)$ then the polynomial equation $f(x)=0$ has a root between a and b.



$f(a)>0$
$f(b)<0$

8) Show that the equation $x^3-16x^2-x-1=0$ has a root which is greater than 8.

=> Let, $f(x)=x^3-16x^2-x-1$

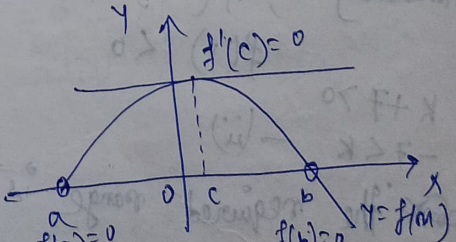
We have, $f(8)=8^3-16\cdot 8^2-8-1=64\times 8-16\times 64-9$

$=512-1026-9=-521<0$

and $f(\infty)>0$.

$\therefore$ By Bolzano's theorem the given equation has a root which is between 8 and infinity that is the given equation has a root which is greater than 8.

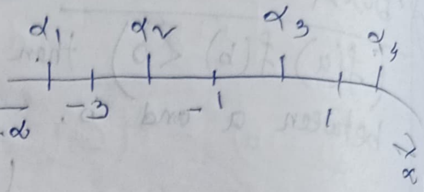
9) Rolle's theorem:- Between any two consecutive roots of the equation $f(x)=0$ there exists at least a root of the equation $f'(x)=0$, where $f(x)$ is a polynomial.



[Note:- The roots of the equation $f(x)=0$ are separated by the roots of the equation $f'(x)=0$.]

10) Find the range of the values of k for which the equation $x^4+4x^3-2x^2-12x+k=0$ has 4 real and unequal roots.

=> Let, $f(x) = x^4+4x^3-2x^2-12x+K$



$\therefore f'(x) = 4x^3+12x^2-4x-12$

$= 4(x^3+3x^2-x-3)$

$= 4(x+3)(x+1)(x-1)$

$\therefore$ The roots of the equation $f'(x)=0$ are $-3, -1$ and 1

We have,

$f(-\infty) > 0$

$f(-3) = 81-108-18+36+K = -9+K$

$f(-1) = 1-4-2+12+K = K+7$

$f(1) = 1+4-2-12+K = K-9$

$f(\infty) > 0$

Since, all the roots of the equation $f'(x)=0$ are real and distinct,

$\therefore$ The roots of the equation $f(x)=0$ has four real roots.

Now, since, $f(-\infty)>0$ and $f(x)=0$ has a root between $-\infty$ and -3,

$\therefore f(-3)<0$

$\therefore K-9<0$

$\therefore K<9$ ——— (i)

Similarly, $f(-1)>0$

$f(1)<0$

$\therefore K+7>0$

or, $-7<K$ ——— (ii)

$\therefore$ ~~hence~~ the required range is $-7<K<9$

11) Multiple roots:- If $f(x)=0$ has a root α, r times then α is called a multiple root of the equation $f(x)=0$ of multiplicity r.

[Note:- If α be a multiple root of the equation $f(x)=0$ of multiplicity r then $f(\alpha)=0, f'(\alpha)=0, f''(\alpha)=0, \ldots, f^{r-1}(\alpha)=0$

12) If the equation $x^4 - 4px^3 + 6x^2 + 1 = 0$ has a multiple root r, Prove that $3p = \frac{r^2+3}{r}$.

$\Rightarrow$ Let, $f(x) = x^4 - 4px^3 + 6x^2 + 1$

$\therefore f'(x) = 4x^3 - 12px^2 + 12x$

Since, r is a multiple root.

$\therefore f(r) = 0$ and $f'(r) = 0$.

Now, $f'(r) = 0$ gives,

$4r^3 - 12pr^2 + 12r = 0$

or, $r(r^2 - 3pr + 3) = 0$

or, $r^2 - 3pr + 3 = 0$ [Since, $r \neq 0$]

or, $3pr = r^2 + 3$

or, $3p = \frac{r^2+3}{r}$ (Proved)

13) If the equation $x^4 + px^2 + qx + r = 0$ has a multiple root of multiplicity 3 then show that $8p^3 + 27q^2 = 0$ and $p^2 + 12r = 0$

$\Rightarrow$ Let, $f(x) = x^4 + px^2 + qx + r$

$\therefore f'(x) = 4x^3 + 2px + q$

$\therefore f''(x) = 12x^2 + 2p$

Let, α be a multiple root of the given equation of multiplicity 3

$\therefore f(\alpha) = 0 \Rightarrow \alpha^4 + p\alpha^2 + q\alpha + r = 0$ — (i)

$\therefore f'(\alpha) = 0 \Rightarrow 4\alpha^3 + 2p\alpha + q = 0$ — (ii)

$\therefore f''(\alpha) = 0 \Rightarrow 12\alpha^2 + 2p = 0$ — (iii)

from (iii), $p = -6\alpha^2$

from (ii), $4\alpha^3 + 2\times(-6\alpha^2)\alpha + q = 0$

or, $q = 12\alpha^3 - 4\alpha^3 = 8\alpha^3$

from (i), $\alpha^4 - \alpha^2 6\alpha^2 + \alpha \cdot 8\alpha^3 + r = 0$

or, $r = 6\alpha^4 - 9\alpha^4 = -3\alpha^4$

$\therefore 8p^3 + 27q^2 = 8(-6\alpha^2)^3 + 27(8\alpha^3)^2 = 0$

$\therefore p^2 + 12r = (-6\alpha^2)^2 + 12(-3\alpha^4) = 0$

14) Show that the equation $x^n + nx^{n-1} + n(n-1)x^{n-2} + \dots + \lfloor n = 0$ can't have a multiple root.

$\Rightarrow$ The given equation can be written as,

$$1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \dots + \frac{x^{n-1}}{\lfloor n-1} + \frac{x^n}{\lfloor n} = 0 \quad \text{--- (i)}$$

Let, $f(x) = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \dots + \frac{x^{n-1}}{\lfloor n-1} + \frac{x^n}{\lfloor n}$

Let, if possible α be a multiple root of the equation $f(x)=0$

Now, $f'(x) = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \dots + \frac{x^{n-1}}{\lfloor n-1}$

Since, α is a multiple root of $f(x)=0$,

$f(\alpha)=0$ and $f'(\alpha)=0$ which give,

$$1 + x + \frac{x^2}{\lfloor 2} + \dots + \frac{x^n}{\lfloor n} = 0 \quad \text{--- (ii)}$$

and $$1 + x + \frac{x^2}{\lfloor 2} + \dots + \frac{x^{n-1}}{\lfloor n-1} = 0 \quad \text{--- (iii)}$$

from (ii) and (iii)

$$\frac{\alpha^n}{\lfloor n} = 0$$

or, $\alpha^n = 0$

or, $\alpha = 0$

which is a contradiction.

$\therefore$ The given equation can't have any multiple root.

11/8/18

15) In an algebraic equation with real coefficient, imaginary roots occurs in conjugate pair.

Consider the equation $f(x) = 0$.

When, $f(x)$ is divided by $\{x-(\alpha+i\beta)\}\{x-(\alpha-i\beta)\}$, the remainder will be of first degree.

Let, $Q(x)$ be the quotient and $(Ax+B)$ be the remainder.

$\therefore$ We have,

$$f(x) = \{x-(\alpha+i\beta)\}\{x-(\alpha-i\beta)\}\, Q(x) + (Ax+B) \quad \text{--- (i)}$$

Let, $\alpha+i\beta$ be an imaginary root of the equation $f(x)=0$.

Now, from (i),

$$A(\alpha+i\beta)+B=0$$

$$\text{or, } (A\alpha+B)+i(A\beta)=0$$

$$\therefore A\alpha+B=0 \quad \text{---(ii)}$$

$$A\beta=0 \quad \text{---(iii)}$$

from (iii), $A=0$ [Since $\beta\neq 0$].

$\therefore$ from (ii), $B=0$

$\therefore$ from (i), $\alpha-i\beta$ is also a root of the given equation.

16) In an equation with rational coefficients, irrational roots occurs in ~~conju~~ conjugate pairs.

17) Prove that the roots of the equation $\frac{1}{x+1}+\frac{1}{x+2}+ \cdots +\frac{1}{x+10}=\frac{1}{x}$ are all real.

$\Rightarrow$ The given equation can be written as,

$$\frac{x}{x+1}+\frac{x}{x+2}+\cdots+\frac{x}{x+10}=1$$

$$\text{or, } \left(\frac{x}{x+1}-1\right)+\left(\frac{x}{x+2}-1\right)+\cdots+\left(\frac{x}{x+10}-1\right)=1-10=0-9$$

$$\text{or, } -\frac{1}{x+1}-\frac{2}{x+2}-\cdots-\frac{10}{x+10}=-9$$

$$\text{or, } \frac{1}{x+1}+\frac{2}{x+2}+\cdots+\frac{10}{x+10}=9 \quad \text{---(i)}$$

Now Let, $(\alpha+i\beta)$ be a root of the equation (i).

$\therefore$ from (i),

$$\frac{1}{\alpha+i\beta+1}+\frac{2}{\alpha+i\beta+2}+\cdots+\frac{10}{\alpha+i\beta+10}=9$$

$$\text{or, } \frac{1}{(\alpha+1)+i\beta}+\frac{2}{(\alpha+2)+i\beta}+\cdots+\frac{10}{(\alpha+10)+i\beta}=9 \quad \text{---(ii)}$$

$\because (\alpha+i\beta)$ is a root of the given equation.

$\therefore (\alpha-i\beta)$ is also a root of the given equation

$\therefore$ from (i), $$\frac{1}{(\alpha+1)-i\beta}+\frac{2}{(\alpha+2)-i\beta}+\cdots+\frac{10}{(\alpha+10)-i\beta}=9 \quad \text{---(iii)}$$

Now,
Subtracting (ii) from (iii),

$$\frac{2i\beta}{(\alpha+1)^2+\beta^2}+\frac{4i\beta}{(\alpha+1)^2+\beta^2}+\cdots+\frac{20i\beta}{(\alpha+1)^2+\beta^2}=0$$

or, $$2i\beta\left[\frac{1}{(\alpha+1)^2+\beta^2}+\frac{2}{(\alpha+1)^2+\beta^2}+\cdots+\frac{10}{(\alpha+1)^2+\beta^2}\right]=0 \quad \text{(iv)}$$

∵ each term within the bracket of L.H.S. of (iv) is positive, the expression is positive and hence from (iv) $\beta=0$.

∴ All the roots of the given equation are real.

18) Obtain the condition that $x^3+3px+q$ may have a factor $(x-a)^2$. [Ans: $a^2(a^2+3p)^2=q^2$]

19) Prove that x^2+x+1 is a factor of $x^{10}+x^5+1$.

20) Show that the equation $x^n-px+r=0$ will have a pair of equal roots if $n^n r^{n-2}=4p^n(n-2)^{n-2}$.

21) If the equation $x^n-qx^{n-m}+r=0$ has two equal roots then $\left\{\frac{q}{n}(n-m)\right\}^n=\left\{\frac{r}{m}(n-m)\right\}^m$.

22) If the equation $x^4+ax^3+bx^2+cx+d=0$ has 3 equal roots, Prove that each of them equal to $\frac{6c-ab}{3a^2-8b}$.

23) If $\alpha_1,\alpha_2,\alpha_3,\alpha_4$ be the roots of the equation $x^4+p_1x^3+p_2x^2+p_3x+p_4=0$ then find the value of $(\alpha_1^2+1)(\alpha_2^2+1)(\alpha_3^2+1)(\alpha_4^2+1)$. ($x=i$, $x=-i$ putting)

24) If $\alpha_1,\alpha_2,\ldots,\alpha_n$ be the roots of the equation $x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots+a_n=0$ then show that $(1+\alpha_1^2)(1+\alpha_2^2)\cdots(1+\alpha_n^2)=(a_1-a_3+a_5-\cdots)^2+(a_1-a_2+a_4-\cdots)^2$